Matrices

Standard notation: \( M_{i,j} \) is the value in row \( i \), column \( j \) of matrix \( M \).

Number of elements: Mathcad numbers the rows and columns of matrices starting with zero. If \( i := 0..imax \), then there are \( imax+1 \) rows in the array.

Origin: If \( \text{Origin}:=1 \) is written, then the counting starts at 1 rather than zero. This statement should be used carefully, if at all.

Typing in a matrix: To create a matrix type the variable name, then open the Matrix toolbar, select the matrix, then the number of rows and columns.

For example,

\[
\begin{bmatrix}
1 & 2 \\
3 & 4 \\
4 & 6 \\
\end{bmatrix}
\]

Then

\[
M_{0,0} = 1 \quad M_{0,1} = 2
\]

\[
M_{1,0} = 3 \quad M_{1,1} = 4
\]

Note that the 1,1 element is NOT the upper left element.

To find the number of rows or columns: \( \text{rows}(M) = 3 \quad \text{cols}(M) = 2 \)

The transpose is \( M^T = \begin{bmatrix} 1 & 3 & 4 \\ 2 & 4 & 6 \end{bmatrix} \)

The reverse: reverse(M) = \( \begin{bmatrix} 4 & 6 \\ 3 & 4 \\ 1 & 2 \end{bmatrix} \)

Column selector \( M^{(i)} = \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix} \)

To create a vector from a row: \( (M^T)^{(i)} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \)

Determinant: \( |N| = -2 \quad |M| = \) This command does not work for a matrix that is not square

Vectorized equations in which the operation is performed term by term (select the vectorize operator from the taskbar):

\[
(N\cdot N) = \begin{bmatrix} 1 & 4 \\ 9 & 16 \end{bmatrix} \quad N^\rightarrow = \begin{bmatrix} 1 & 1 \\ \end{bmatrix}
\]

Multiplication by a scalar \( 5\cdot N = \begin{bmatrix} 5 & 10 \\ 15 & 20 \end{bmatrix} \quad N_{1,1} \cdot N = \begin{bmatrix} 4 & 8 \\ 12 & 16 \end{bmatrix} \)

Matrix inverse:
(to solve a set of linear equations) If \( N = \begin{bmatrix} 7 & 25 \\ 9 & 57 \end{bmatrix} \) then \( N^{-1} = \begin{bmatrix} 25 & 7 \\ 57 & 9 \end{bmatrix} \)
Making matrices larger or smaller

On the previous page we defined: 

\[
M = \begin{pmatrix} 
1 & 2 \\
3 & 4 \\
4 & 6 
\end{pmatrix}, \quad N = \begin{pmatrix} 
1 & 2 \\
3 & 4 
\end{pmatrix}
\]

**Stack** is a command that adds the rows of the second argument to the rows of the first argument.

\[
\text{stack}(M, N) = \begin{pmatrix} 
1 & 2 \\
3 & 4 \\
4 & 6 \\
1 & 2 \\
3 & 4 
\end{pmatrix}
\]

The arguments must have the same number of columns.

**Stack with programs:** Stack can be used to return more than one value from a program.

\[
A := \begin{align*} 
a &\leftarrow 1 \\
b &\leftarrow e \\
c &\leftarrow \pi \\
\text{stack}(a, b, c) 
\end{align*}
\]

\[
A = \begin{pmatrix} 
1 \\
2.718 \\
3.142 
\end{pmatrix}
\]

**Augment** is a command that places the columns of the second argument after the columns of the first argument.

\[
\text{augment}(N, M^T, N) = \begin{pmatrix} 
1 & 2 & 1 & 3 & 4 & 1 & 2 \\
3 & 4 & 2 & 4 & 6 & 3 & 4 
\end{pmatrix}
\]

There can be more than 2 arguments. The arguments must have the same number of rows.

**Submatrix** is used to select part of a matrix:
The five arguments are the matrix, the start and end rows, and the start and end columns.

\[
M = \begin{pmatrix} 
1 & 2 \\
3 & 4 \\
4 & 6 
\end{pmatrix}
\]

\[
\text{submatrix}(M, 1, 2, 0, 1) = \begin{pmatrix} 
3 & 4 \\
4 & 6 
\end{pmatrix}
\]

\[
\text{submatrix}(M, 1, 2, 0, 0) = \begin{pmatrix} 
3 \\
4 
\end{pmatrix}
\]

We have selected rows 1 and 2 and columns 0 and 1.

We have selected rows 1 and 2 and column 0.