Reducing noise by signal averaging

It is possible to increase the signal-to-noise ratio by averaging signals if the signals are repetitive or periodic. The key insight is that *noise is random* and the sum of N noise signals will be larger by only a factor of the square root of N. If the signal is repetitive and the timing of the signals is known and used, then the signals will add linearly and the sum will be larger by a factor of N. Thus N measurements will increase the signal to noise ratio by a factor of the square root of N.

We will demonstrate this by adding N = 100 signals that have a small repetitive sinusoidal signal and random noise.

I. The digital oscilloscope

A modern digital oscilloscope samples a signal at equal intervals Δt and the voltages are put into a vector V_k with a length of kmax samples. For some oscilloscopes kmax = 2500. This means that if the oscilloscope is set at 5 ms per small division, the full time for a sweep (10 divisions) is 50 ms, and the time interval between samples is Δt = 50 ms / 2500 samples = 0.02 ms = 20 microseconds.

II. The signal

$kmax := 2499$ The number of sampled voltages in one oscilloscope trace is 2500.

$k := 0 .. kmax$ A subscript for numbering the voltage samples.

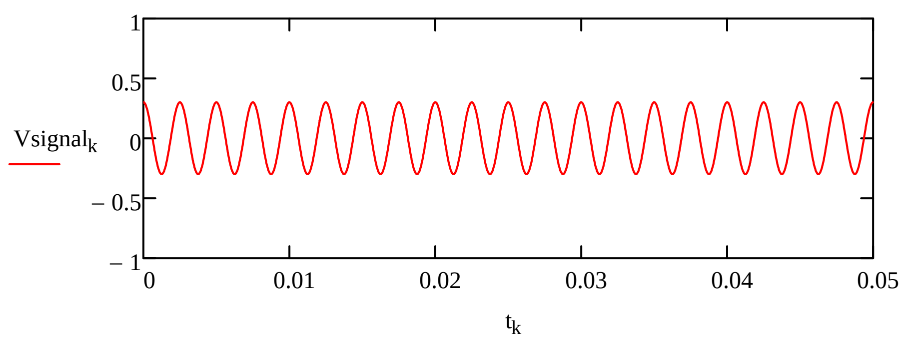
$\Delta t := 20 \cdot 10^{-6}$ The interval, 20 μs, between oscilloscope samples.

$t_k := \Delta t \cdot k$ The time of the kth sample taken by the osciloscope.

$f := 400$ The frequency of our sinusoidal voltage (the signal).

$Vsignal_k := 0.3 \cdot \cos(2 \cdot \pi \cdot f \cdot t_k)$ The output of a digital oscilloscope that is measuring our signal if there is no noise.

The signal as a function of time



III. The noise

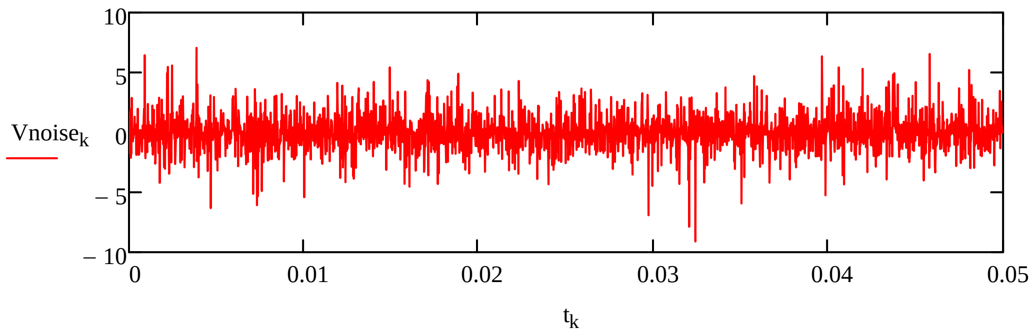
Random noise will be made using the **rnd** function built into Mathcad. The function rnd(1) returns a number chosen randomly on the interval 0,1. We then take the natural log of the random numbers to generate numbers that are distributed exponentially. To have both positive and negative numbers, we will multiply by the factor sign(rnd(1)-0.5). The argument of the sign function generates random numbers between 0.5 and -0.5, thus for half of the arguments the sign returns +1 and for the other half it returns -1.

Try it: Generate a histogram (using the hist function) of the numbers generated by ln(rnd(1)) and convince yourself that the distribution of numbers is exponential.

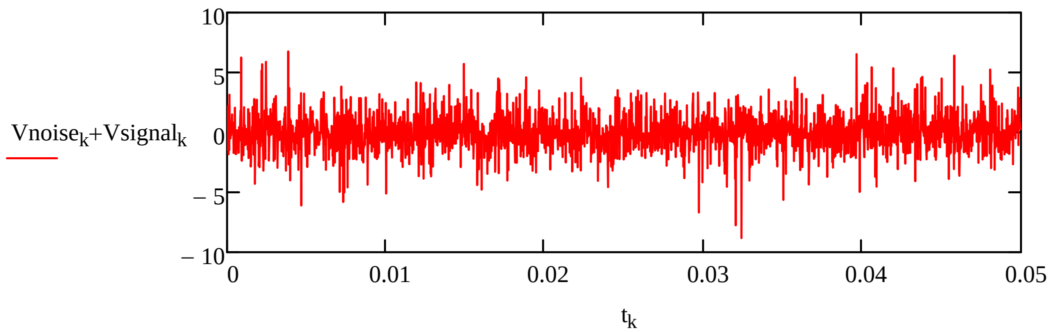
$Vnoise_k := \mathrm{sign}(\mathrm{rnd}(1) - 0.5)\cdot\ln(\mathrm{rnd}(1))$ This is the function to generate the random noise.

The function rnd(1) generates a new random number every time it is called.

The noise as a function of time



This is the signal and the noise combined. Can you see the signal?



III. Reducing noise by averaging

Now let's generate 100 oscilloscope traces that are the signal plus the noise. We will put the traces into a matrix for averaging. The matrix $V_{j,k}$ will have k = 2500 samples in each row. There will be j = 100 rows.

$jmax := 99 \qquad j := 0 .. jmax$ The number of oscilloscope traces to average is 100.

$V_{jmax,kmax} := 0$ Initialize the matrix to zero.

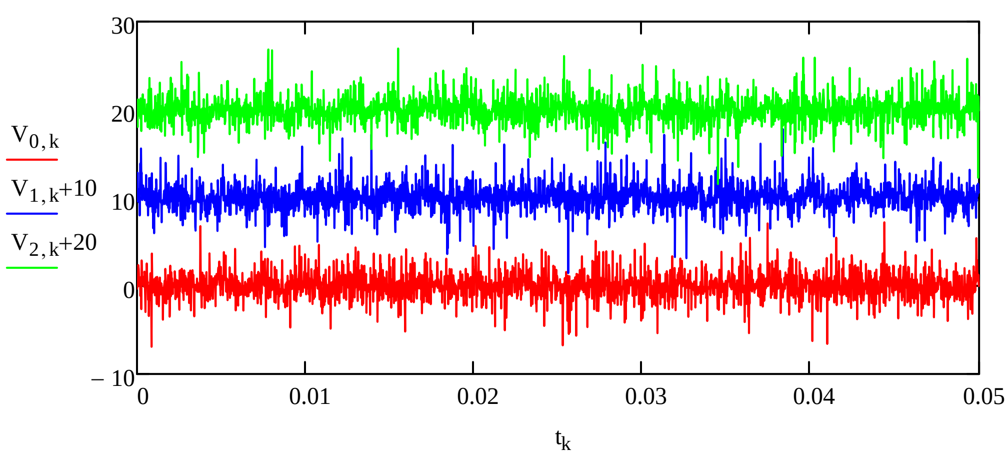
$rows(V) = 100 \qquad cols(V) = 2500$ This verifies the size of the matrix.

The line below generates 100 oscilloscopes traces with signal plus noise. Note that new noise is generated in each traces and that we are not adding together identical noise traces.

$$V_{j,k} := Vsignal_k + sign(rnd(1) - 0.5) \cdot \ln(rnd(1))$$

Plotted below are three consecutive traces of signal + noise. The traces have been offset by 10 V so that they do not overlap. Can you detect the signal visually?

Three noisy signals

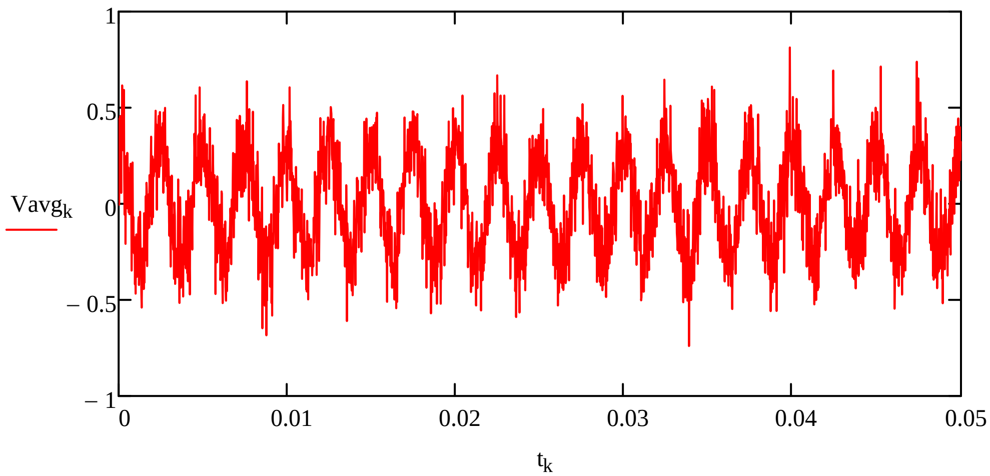


Apply signal averaging:

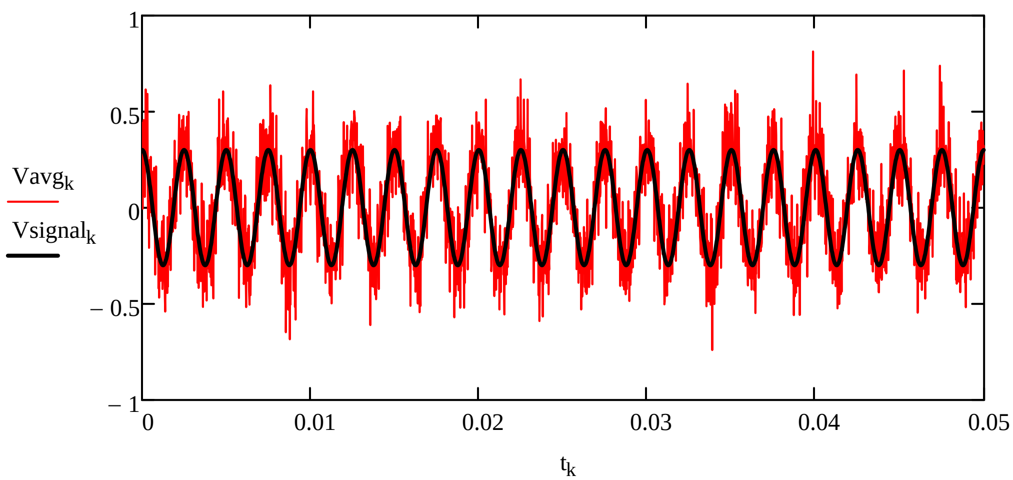
Now average the 100 signals by adding the signals and dividing by 100:

$$Vavg_k := \frac{1}{jmax + 1} \cdot \sum_{j=0}^{jmax} V_{j,k}$$

The average of 100 noisy traces



The average of 100 noisy traces plotted together with the signal



The signal averaging has recovered the signal sufficiently from the noise for us to determine the frequency and the amplitude of the sine wave by visual inspection.

Try it: How would our results be different if the noise were identical in each trace?

Try it: Is the signal easier to see if the number of signals averaged is 1000 (jmax=999)?

Try it: The noise in the averaged signal is smaller than the noise in the unaveraged signals. By what factor is the noise reduced?