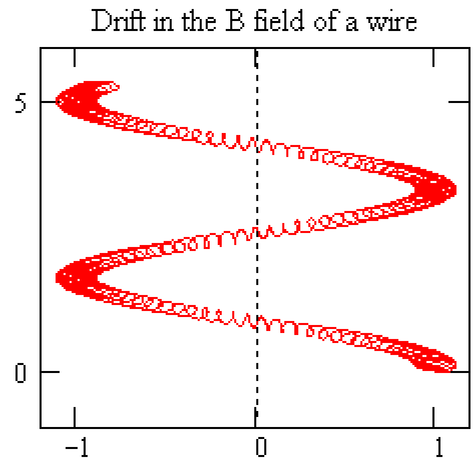
Drifts in the field of a wire



We will use Runge-Kutta on the equations of motion to follow a charged particle in the B field of a wire. The field of a wire falls inversely with radius so this simple geometry gives us a magnetic field that varies in space.

ORIGIN := 1

Our 6-vector for the particle coordinates will have subscripts 1,2 and 3 for x,y, and z and 4,5, 6 for Vx, Vy, Vz.

$$Z := \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad \begin{matrix} X \\ Y \\ Z \\ Vx \\ Vy \\ Vz \end{matrix}$$

$I := 50$ 50 units is the current. There will create B_θ and $B_z = 0$.

We will let $\mu_0 = 1$ so that our answers do not require scientific notation: $\mu_0 := 1$

In terms of the components of Z, the radius is $r(Z) := \sqrt{\left(Z_1\right)^2 + \left(Z_2\right)^2}$

The θ field is $B_\theta(Z) := \dfrac{\mu_0 \cdot I}{2 \cdot \pi\, r(Z)}$

The θ field in x,y,z coordinates is

$$B(Z) := \begin{bmatrix} \dfrac{-\left[(Z)_2 \cdot B_\theta(Z)\right]}{r(Z)} \\ \dfrac{Z_1 \cdot B_\theta(Z)}{r(Z)} \\ 0 \end{bmatrix} \quad \begin{matrix} \text{Components of B} \\ Bx \\ By \\ Bz \end{matrix}$$

Test this, at x = 1 we get:

$$B(Z) = \begin{pmatrix} 0 \\ 7.958 \\ 0 \end{pmatrix}$$

On the x axis at x = 1, the B field is along y as we expect.

Find and plot the particle trajectory

Pick starting X and V vectors $X := \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ $V := \begin{pmatrix} 0.9 \\ 0.1 \\ 0 \end{pmatrix}$

V is nearly perpendicular to B, which is along y at our starting point.

We will stack these values and put them into our 6-vector z $Z := \text{stack}(X, V)$

The next two definitions let us recover X and V from the stack Z:

$X(Z) := \text{submatrix}(Z, 1, 3, 1, 1)$ $V(Z) := \text{submatrix}(Z, 4, 6, 1, 1)$

The submatrix command is used above to create a subvector.

We will let q and m =1 to avoid scientific notation: $q := 1$ $m := 1$

DZ is the derivative of the 6-vector Z. The bottom three terms, the accelerations, are found from the Lorentz force.

$$DZ(t,Z) := \begin{bmatrix} Z_4 \\ Z_5 \\ Z_6 \\ \left(\frac{q}{m}\cdot V(Z) \times B(Z)\right)_1 \\ \left(\frac{q}{m}\cdot V(Z) \times B(Z)\right)_2 \\ \left(\frac{m}{q}\cdot V(Z) \times B(Z)\right)_3 \end{bmatrix}$$

dx/dt
dy/dt
dz/dt

dVx/dt
dVy/dt
dVz/dt

The gyro frequency is

$$\Omega := \frac{q}{m}\cdot B(Z)_2$$

$$\Omega = 7.958$$

Let t be the total time interval $t := 100$

Our time interval in the Runge Kutta should be divided more finely than the gyration period.

The number of iterations we need is about 4 per gyro period or 8 π per orbit.

$npoints := ceil(4\cdot|\Omega|\cdot t)$ $npoints = 3.184 \times 10^3$ (ceil is a rounding function to make integers)

Now integrate: $F := rkfixed(Z, 0, t, npoints, DZ)$

t x y z Vx Vy Vz

F =

	1	2	3	4	5	6	7
1	0	1	0	0	0.9	0.1	0
2	0.031	1.028	$3.14\cdot10^{-3}$	$3.482\cdot10^{-3}$	0.873	0.1	0.22
3	0.063	1.054	$6.275\cdot10^{-3}$	0.014	0.796	0.1	0.421
4	0.094	1.078	$9.39\cdot10^{-3}$	0.03	0.676	0.099	0.594
5	0.126	1.096	0.012	0.051	0.523	0.097	0.733
6	0.157	1.11	0.015	0.075	0.345	0.095	0.832
7	0.188	1.118	0.018	0.102	0.151	0.092	0.888
8	0.22	1.12	0.021	0.131	-0.049	0.088	0.9
9	0.251	1.115	0.024	0.158	-0.247	0.084	...

Click and scroll to see the rest of the values.

Let's create a special file that contains the wire location so it can be plotted also. The wire is along the z axis. The wire can be represented by increasing numbers along the z axis column of a matrix called "Wire" that has the same columns as F above.

$k := 1..40 \qquad l := 1..7 \qquad Wire_{k,1} := 0 \qquad Wire_{k,4} := \frac{k}{5} - 1$

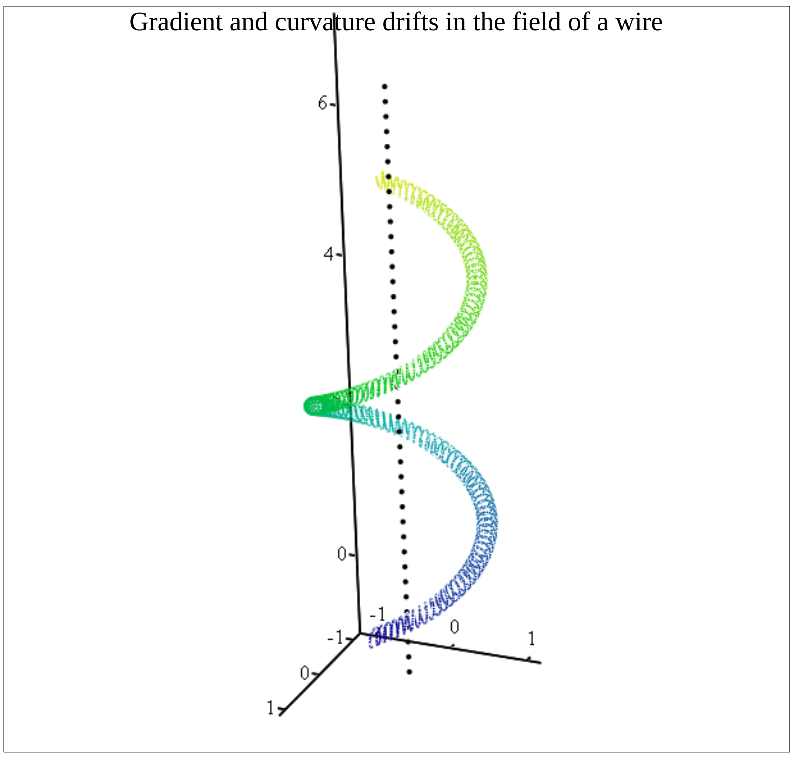
x y z

Wire =

	1	2	3	4	5	6	7
1	0	0	0	-0.8	0	0	0
2	0	0	0	-0.6	0	0	0
3	0	0	0	-0.4	0	0	0
4	0	0	0	-0.2	0	0	...

Click and scroll to see that z goes from -0.8 to +7.

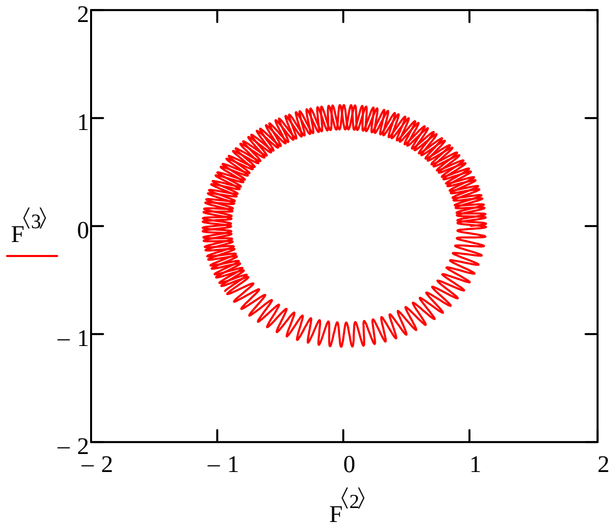
An interactive 3-d view of the wire and the particle, grab a corner and pull:



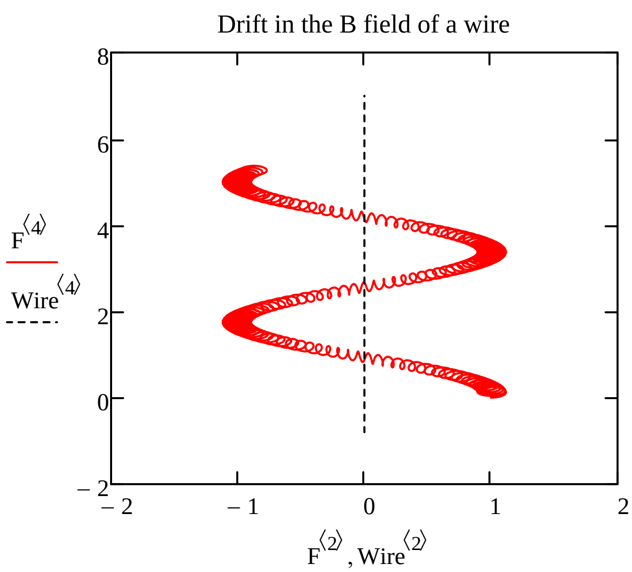
$\left(F^{\langle 2\rangle}, F^{\langle 3\rangle}, F^{\langle 4\rangle}\right), \left(Wire^{\langle 2\rangle}, Wire^{\langle 3\rangle}, Wire^{\langle 4\rangle}\right)$

This is a 3-d scatter plot of two matrices, F and Wire. The particle is spiralling in little circles around the field lines, which encircle the wire (the black dots in the middle). Because of the magnetic gradient, there is a **gradient drift** in the z direction, parallel to the wire.

This view looks down from above the wire. It is the x,y plane:



Below is a side view:



x

The lines of B are simply circles around the wire. The particle trajectory is helical because of the graient drift along z.

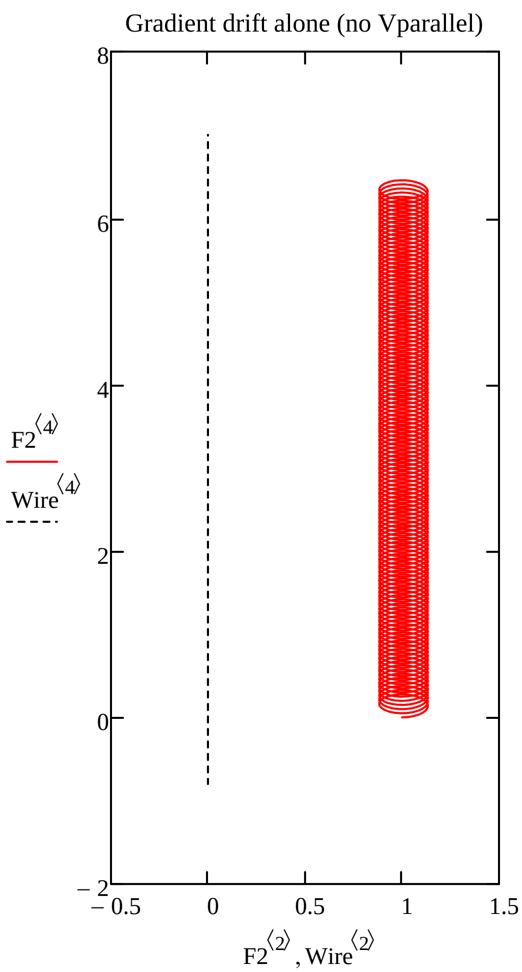
Curvature drift or gradient drift?

The particle above was given initial velocity along both x and y. The x velocity is perpendicular to B at the starting location and the y velocity is parallel. The curvature drift is proportional to the parallel velocity squared. So the curvature drift will go away if we only use perpendicular velocity. Let's plot the trajectory for initial velocity perpendicular to B.

$$Z := \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

Now we are starting at x = 1 with $v_x = 1$.
On the x axis B is in the y direction so this v is perpendicular to B.

Solve for the trajectory: $F2 := \text{rkfixed}(Z, 0, t, npoints, DZ)$



The guiding center drifts upward in z. There is no motion along the field line which is in the θ direction.

This is the gradient drift only.

The wire is on the left.

Curvature drift only

Let's start again with velocity just along the B field, which points in the y direction at our starting point x. It goes almost twice as far at the example above. That is because the combined drifts are proportional to

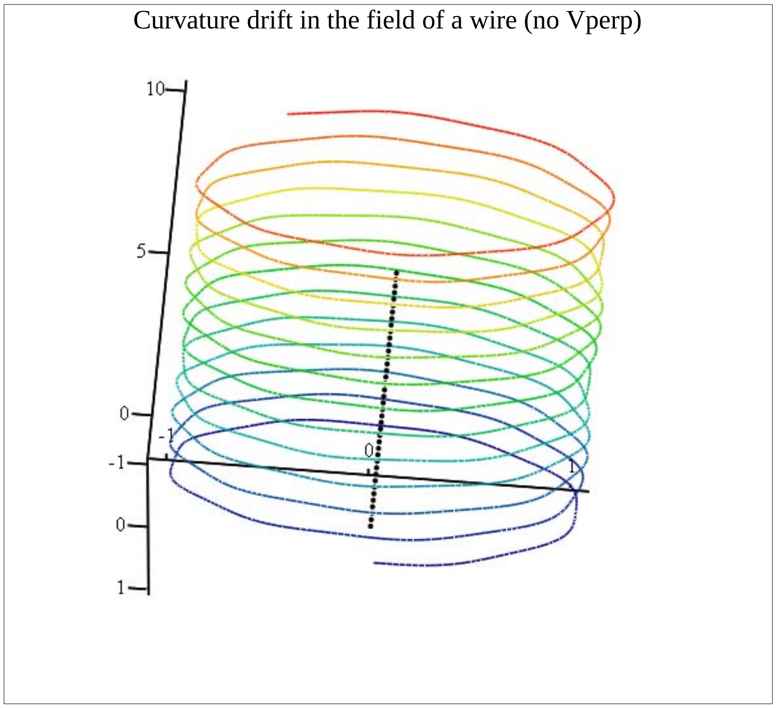
$$Z := \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

$$\tfrac{1}{2} v_{\perp}^2 + v_{\parallel}^2$$

where the subscripts on *v* refer to the directions perpendicular and parallel to the B field.

Solve for the trajectory:

$$F3 := \text{rkfixed}(Z, 0, t, \text{npoints}, DZ)$$



$$\left(F3^{\langle 2\rangle}, F3^{\langle 3\rangle}, F3^{\langle 4\rangle}\right), \left(\text{Wire}^{\langle 2\rangle}, \text{Wire}^{\langle 3\rangle}, \text{Wire}^{\langle 4\rangle}\right)$$

This is a 3-d plot with line and color map selected for plot 1, the particle trajectory.
Points are selected for plot 2, the central wire.

With motion along B, the Larmor radius is nearly zero and we see the curvature drift but only a hint of the gyro motion. This spiral encircles the wire and is NOT gyro motion. The gyro motion is the little spirals in the previous graph.

Question: What is the Larmor radius in this plot?