Langmuir Probe Data Analysis

A Langmuir probe is simply a wire (or disk) placed in the plasma. The probe collects mostly electrons when biased positively and mostly ions when biased negatively. When the bias is negative, the electrons that are collected are the ones with sufficient energy to overcome the potential barrier of the probe. In this exercise we will analyze data from a cylindrical Langmuir probe. The random current of electrons to a surface in a plasma is

$$J_{sat} = n_e q \sqrt{kT_e / 2\pi m_e}$$

where J_{sat} is the current density, n_e is the electron density, m_e is the electron mass, and q is the elementary charge. The surface is assumed to be at the plasma potential. The electron temperature T_e will be written in electron volts below, which means that the thermal energy will be qT_e rather than kT_e. When the probe is at the plasma potential, the current collected is the **saturation current** I_{sat}**:**

$$I_{sat} = A_p J_{sat} = 2\pi a L J_{sat}$$

where a is the probe radius, L is the probe length and A_p is the probe area.

When the probe voltage is negative relative to the plasma potential, Vplasma, the current is:

$$I(V) = I_{sat} \exp[(V - V_{plasma})/T_e]$$

Vplasma is the potential in the space between plasma electrons and ions.

Characteristics of the probe used to acquire the data:

$a := 95 \cdot 10^{-6}$ Probe radius in m. The wire diameter is 0.0075 inches.

$L := 0.027$ Probe length in m. The length is approximately 1 inch.

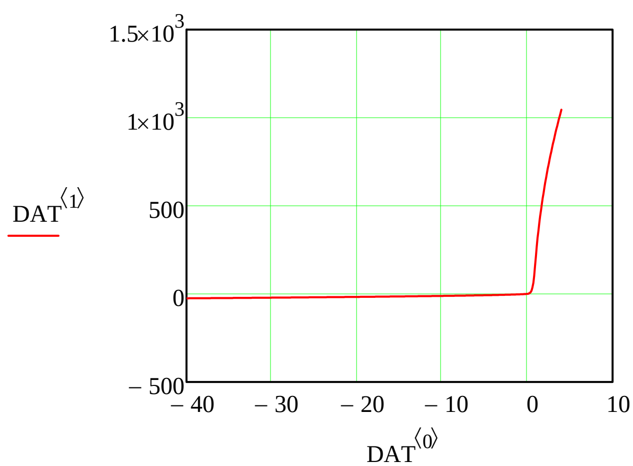
$Ap := 2 \cdot \pi \cdot a \cdot L$ $Ap = 1.612 \times 10^{-5}$ Probe surface area in m^2.

This is the data file imported from a text file:

DAT :=

	0	1
0	-40	-25.95
1	-39.9	-25.98
2	-39.8	-25.93
3	-39.7	-25.89
4	-39.6	-25.9
5	-39.5	-25.86
6	-39.4	-25.83
7	-39.3	-25.78
8	-39.2	...

Probe current in microamps versus probe voltage:



$rows(DAT) = 441$ Number of rows in the file.

$DatRows := rows(DAT) - 1$ The last data row is numbered DatRows.

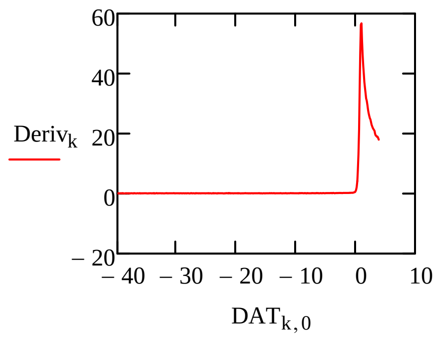
1. Find the plasma potential

The plasma potential is located at the maximum in the first derivative of the current. We find this derivative from the difference between the next current data point and the previous one.

$k := 1 .. DatRows - 1$

$Deriv_k := 0.5 \cdot \left(DAT_{k+1,1} - DAT_{k-1,1} \right)$

Because we need the difference between the data at k+1 and at k-1, our counter k must begin and end one point from the ends. (There is no k= -1 data point.)



$\max(Deriv) = 56.706$ Find the maximum.

Use the **match** function to find the index.

$Location := match(\max(Deriv), Deriv)$

$Location = (410)$

Location is returned as a vector of one element (note the parentheses).
We must change it to an integer.

$Location := Location_0$ Select the zeroth element of the vector.

$Location = 410$ The index corresponding to the maximum derivative.

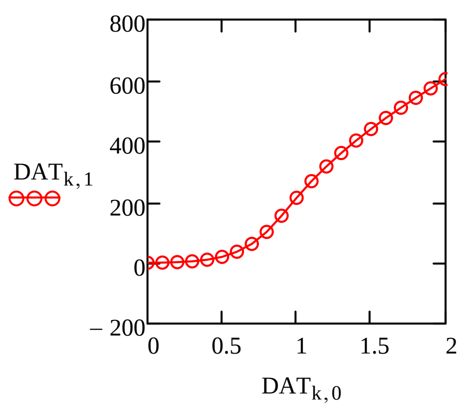
$DAT_{Location,0} = 1.000$ Probe voltage at the plasma potential, Vplasma.

$DAT_{Location,1} = 213.298$ Probe current at the plasma potential.
This is the saturation current Isat.

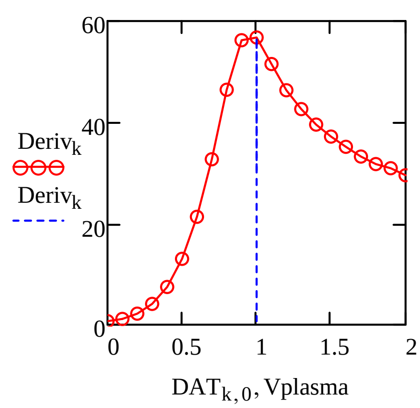
$Isat := DAT_{Location,1}$ $Vplasma := DAT_{Location,0}$ Define Isat and Vplasma.

$k := Location - 10 .. Location + 10$ Define a region near the plasma potential for plotting.

The probe current near the plasma potential:



The derivative of the probe current with Vplasma indicated by a line:



2. Find the temperature from the slope of the logarithm of the current

Create a subset of 5 data points to the left of the plasma potential.

$$\text{Sub} := \text{submatrix}(\text{DAT}, \text{Location} - 5, \text{Location} - 1, 0, 1)$$

$$\text{Sub} = \begin{pmatrix} 0.5 & 19.132 \\ 0.6 & 35.636 \\ 0.7 & 61.702 \\ 0.8 & 100.916 \\ 0.9 & 154.464 \end{pmatrix}$$

Create a vector logI with the logarithms of the currents:

$$\text{logI} := \overrightarrow{\ln\left(\text{Sub}^{\langle 1 \rangle}\right)}$$

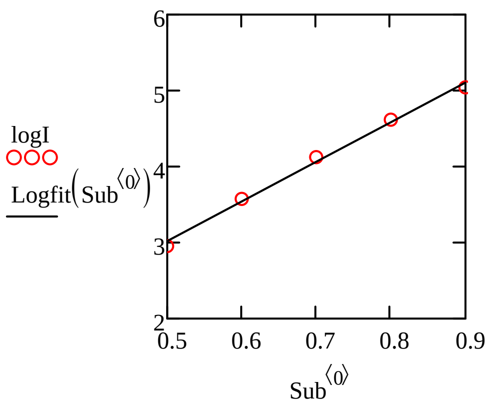
The **vectorize** command (arrow on top) above the ln command creates a new matrix whose elements are the $\log_e$ of the elements of the original vector.

Now we can use the **line** function to return the slope and intercept.

$$\text{AB} := \text{line}\left(\text{Sub}^{\langle 0 \rangle}, \text{logI}\right) \qquad \text{AB} = \begin{pmatrix} 0.408 \\ 5.218 \end{pmatrix} \begin{matrix} \text{intercept} \\ \text{slope} \end{matrix}$$

$$\text{Logfit}(x) := \text{AB}_0 + \text{AB}_1 \cdot x$$

We check the quaility of the fit by plotting the data and the fitted function:



Now we can find Te from the inverse of the slope of the log graph:

$$\text{Te} := \frac{1}{\text{AB}_1}$$

$\text{Te} = 0.192$ Temperature in eV.

3. Find the density from Isat and the thermal velocity of electrons

Physical constants: $q := 1.6 \cdot 10^{-19}$ $\quad \text{me} := 9.11 \cdot 10^{-31}$

Calculate the "flux" velocity of the electrons:

$$\text{ve} := \sqrt{\frac{q \cdot \text{Te}}{2\pi \cdot \text{me}}}$$

Note that qT_e is the electron energy because our temperature is in eV. We can use T_e and Isat and the equation at the beginning of the exercise to find the electron density n_e.

$$\text{ne} := \frac{\text{Isat} \cdot 10^{-6}}{\text{ve} \cdot \text{Ap} \cdot q} \qquad \text{ne} = 1.13 \times 10^{15}$$

Electron density in m^{-3}.

Now we have found the electron density and temperature from the probe data.
Note that Isat has been converted from microamps to amps.

A more sophisticated analysis:

1. Find the plasma potential from the maximum in the slope of a fitted curve

Our method of finding the plasma potential chooses the data point nearest the maximum in the slope. Suppose the maximum is between data points. We could find the maximum more accurately using a smooth curve fit to the data. A maximum in the first derivative is a zero of the second derivative. A cubic polynomial is the lowest order polynomial which has a second derivative that varies, so we will fit a cubic polynomial to the data.

First we create a submatrix of 7 data points near the maximum.

$$W := \text{submatrix}(DAT, Location - 3, Location + 3, 0, 1)$$

$$W = \begin{pmatrix} 0.7 & 61.702 \\ 0.8 & 100.916 \\ 0.9 & 154.464 \\ 1 & 213.298 \\ 1.1 & 267.875 \\ 1.2 & 316.243 \\ 1.3 & 360.433 \end{pmatrix}$$

This is the data submatrix.

$$Fit(x) := \begin{pmatrix} x^3 \\ x^2 \\ x \\ 1 \end{pmatrix}$$

The function **linfit** will fit linear combinations of these functions and return the coefficients.

The zero of the second derivative is at -b/3a if the polynomial is $ax^3 + bx^2 + cx + d = 0$.

$$ss := \text{linfit}\left(W^{\langle 0 \rangle}, W^{\langle 1 \rangle}, Fit\right)$$

The function linfit will find the coefficients of the terms.
This function will work without initial guesses for the values.

$$ss = \begin{pmatrix} -833.528 \\ 2.489 \times 10^3 \\ -1.905 \times 10^3 \\ 460.611 \end{pmatrix}$$

This is the matrix containing the coefficients a,b,c,d.

$$Vplasma := \frac{-ss_1}{3 \cdot ss_0}$$

$$Vplasma = 0.995$$

Vplasma, interpolated value

This value of Vplasma does not have to lie precisely on a data point.

2. Shift the data so that the plasma potential is at zero

$$DAT2 := DAT$$

Put the data in a new array before shifting.

$$DAT2^{\langle 0 \rangle} := \overrightarrow{\left(DAT^{\langle 0 \rangle} - Vplasma\right)}$$

The **vectorize** command subtracts Vplasma from each element of the first column of the new data array.

DAT2 is a new data set that has the plasma potential shifted so that Vplasma is at zero. If we do NOT do this, the current varies as exp(V-Vplasma)/Te. After the shift, the current varies as exp(V/Te).

3. Fit the ion current to the data more negative than -10 volts

At very negative probe voltages, the current collected is from ions. We can fit a model to the data at negative probe voltages and then subtract the model ion current from the total current to get the electron current alone. First we must find where the probe voltage passes -10 V. We use a **program loop** searching the data downward and we leave the loop at the first probe voltage below -10 V.

$$k10 := \left| \begin{array}{l} j \leftarrow DatRows - 1 \\ \text{while } DAT2_{j,0} > -10 \\ \quad j \leftarrow j - 1 \end{array} \right.$$

k10 will be the value of the index where the probe voltage ppasses -10 V.

$k10 = 309$ $DAT2_{k10,0} = -10.095$

$DAT3 := \text{submatrix}(DAT2, 0, k10, 0, 1)$

This creates a subset of the data containing data for voltages below -10 V. This subset is DAT3.

Theoretical analysis (see ref. 1 below) suggests that the ion current should be fit by two functions. One that varies linearly with voltage and one that varies as the square root of the absolute value of the voltage. The fitting function is:

$$Fit2(x) := \begin{pmatrix} \sqrt{-x} \\ x \end{pmatrix}$$

where x will be the probe voltage, and the voltage will be negative.

$coeff2 := \text{linfit}\left(DAT3^{\langle 0 \rangle}, DAT3^{\langle 1 \rangle}, Fit2\right)$

The function **linfit** finds the two coefficients.

$$coeff2 = \begin{pmatrix} -3.429 \\ 0.102 \end{pmatrix}$$

coeff2 are the coefficients that best fit the data.

$Iion(x) := coeff2_0 \cdot \sqrt{-x} + ss2_1 \cdot x$

This is the function that is fit to the ion current.
x is a dummy variable that becomes the voltage.

$Iion(x) := coeff2 \cdot Fit2(x)$

This **dot product** definition for the fitted ion current is neater than the line above, which has been disabled.

$x := DAT_{0,0} - 2, DAT_{0,0} .. 0$

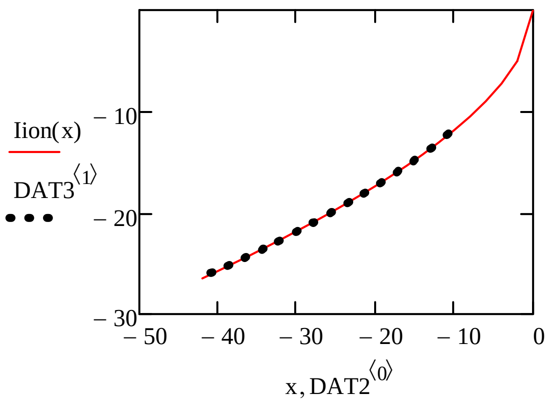
Here we define a range of probe voltages to put in the formula for Iion(x). This goes from below the starting voltage in the data to voltage zero counting by twos.

$DAT_{0,0} = -40$

The data set that we are using starts at -40 volts.

We do not try to fit the ion current for voltages more positive than -10 V, because the current at these voltages has a significant electron contribution.

Here we look at the quality of the fitted function for the ion current:



The quality of the fit is good. The fitted function is the solid line and the data are the dotted line. The graph popup box has been used to adjust the dot size.

We will not use the subset DAT3 again.

4. Subtract the ion current to find the current of electrons alone

Now we subtract the fitted ion current from the total current for all the points to the left of the plasma potential. No ions are collected when the probe is positive, so there is no need to subtract the function for positive voltages.

$$k0 := \left| \begin{array}{l} j \leftarrow DatRows - 1 \\ \text{while } DAT2_{j,0} > 0 \\ \quad j \leftarrow j - 1 \end{array} \right.$$

This **program loop** finds the index where the probe voltage passes zero. We use the full data set DAT2 rather than the subset DAT3. Recall that k0 is the index closest to zero probe voltage.

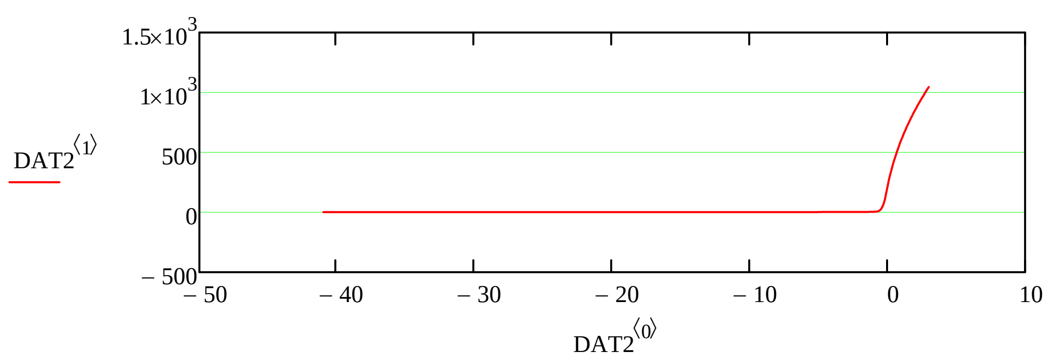
$k0 = 409$ $\quad DAT2_{k0,0} = -0.095$ Verify that k0 is the index near zero probe voltage.

$k := 0 .. k0$ This is the range of k where we subtract the ion current.

$$DAT2_{k,1} := DAT2_{k,1} - Iion\left(DAT2_{k,0}\right)$$

We cannot use the vectorize function for this because we do not want to do the subtraction at positive probe voltages. The line above this defines the range of k for which the operation is performed.

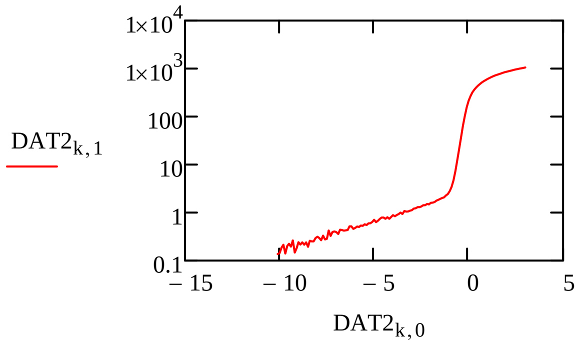
Electron current to the probe as a function of bias voltage:



Zoom in on the most interesting parts of the plot:

The electron current of interest is from -10 V to the positive end. Plot only that part of the data:

$k := k10 .. \text{rows}(DAT) - 1$ k10 was defined as the index where the voltage passes -10.



The electron distribution has a long high energy tail. We will try to fit the data to two Maxwellian distributions, one with a higher temperature than the other.

5. Fit two Maxwellian distributions to the probe current

We will try to fit the electron current to the following function:

$$F(V, Isat, Te, Isat2, Te2) := Isat \cdot \exp\left(\frac{V}{Te}\right) + Isat2 \cdot \exp\left(\frac{V}{Te2}\right)$$

Isat2 is the saturation current of the high energy tail and Te2 is the temperature of the high energy tail. From our simpler analysis at the beginning, we have already found out that:

$Isat = 213.298$ This is in microamps.

$Te = 0.192$ This is in electron volts.

As a first guess we will try: $Isat2 := \frac{Isat}{100}$ $Te2 := 10Te$

The minimize function:

The minimize function will find the values of variables that minimize the value of a function. We will define a new function Err(Isat,Te,Isat2,Te2) that is the difference between the fitted function, F(V,Isat,Te,Isat2,Te2), and the data. At very negative voltages, the current is small, and the error in the fit will not contribute much to the overall error. To make all the data points count about equally in the fit, we will use the difference between the logarithm of the function and the logarithm of the data. The error is measured by summing the squares of the differences.

$k := k10 .. k0$ This is the range of k for which the function must fit the data.

The function Err is the sum of the squares of the errors:

$$Err(Isat, Te, Isat2, Te2) := \sum_k \left(\ln\left(DAT2_{k,1}\right) - \ln\left(F\left(DAT2_{k,0}, Isat, Te, Isat2, Te2\right)\right)\right)^2$$

$P := Minimize(Err, Isat, Te, Isat2, Te2)$ The **minimize** function finds the best values for Isat, Isat2, Te, and Te2. This function, unlike linfit, requires initial guesses for the variabiles.

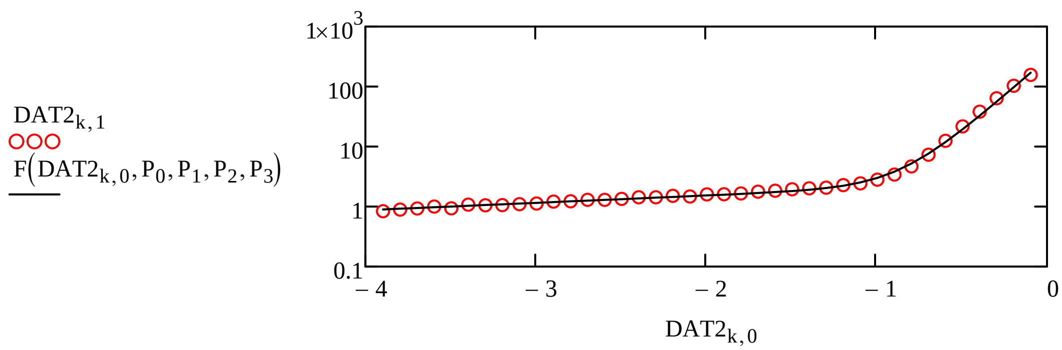
$$P = \begin{pmatrix} 288.517 \\ 0.175 \\ 2.664 \\ 3.556 \end{pmatrix}$$

Isat
Te
Isat2
Te2

The vector P that is returned by the minimize function gives us the values for the parameters that best fit the data.

The plot below shows that we have obtained a good fit. The circles are the data and the line is the fit.

$k := k10 + 62 .. k0$ Plot the data over a small range.



6. Find the density and temperature from the improved parameters

Redefine the plasma parameters to be the newer values:

$Te := P_1$ $Isat := P_0$ Recall that Isat in in microamps

Put the new values in the formulas:

$$ve := \sqrt{\frac{q \cdot Te}{2\pi \cdot me}} \qquad ne := \frac{Isat \cdot 10^{-6}}{ve \cdot Ap \cdot q}$$

The final plasma parameters are:

$Te = 0.175$ Electron temperature in eV.

$ne = 1.6 \times 10^{15}$ Electron density in m^{-3}.

Most of the electrons in our plasma have a temperature of 0.175 eV. There is a high energy tail on the distribution with temperature 3.5 eV that is secondary electrons emitted from the walls.

Reference: "Langmuir probe interpretation for plasmas with secondary electrons from the wall," Z. Sternovsky and S. Robertson, Physics of Plasmas 11(7), pp. 3610-3615, July 2004.